1 A real plasma

1.1 Introduction

In our previous lesson, we have treated the two key parts of a fluid plasma model: the particle balance and the energy balance. [1]. Today, we will gradually combine these balances to create a full model of a plasma.

1.2 `example3.ccp`

Simply put,`example3.ccp` is a possible implementation of the exercises of the previous lesson. Note, however, that we have included the underrelaxation factors. We will briefly discuss it.

We begin with a large number of constant declaration. Then, we get the function that computes the buffer density. After that, some transport coefficients are computed: ambipolar diffusion and recombination. Then, in the `main` function, several fields are created: Fields for the electron and heavy particle temperatures, and fields for the neutral and ion densities. After that, we see the initialization loop. Then, the underrelaxation factors are defined. The next part is the main iteration loop. First, the transport coefficients and sources are updated. Then, the matrix equations are solved. Finally, the results are written to log.

1.3 Adding the electron temperature

We will start the completion of our model by including the electron temperature. The energy contained in the hot electrons is essential for the sustaining of virtually all plasmas. They are the ones that create the ionizations that replenish the ions lost by diffusion or three-particle recombination. Our goal will be to describe the ionization, the recombination and the heat conductivity of the electrons. The first two are important both for the particle reactions and for the electron energy balance.

Exercise 1 *Write a function that given an electron temperature and neutral density returns the ionization rate per electron (if you would multiply the result with the electron density, you would get the ionization frequency per cubic meter.) The rate coefficient is* $1 \cdot 10^{-15}$ $m^3 s^{-1}$. *Use this rate coefficient as the source term for the particle balance. Use a linear profile of the electron temperature, with a central temperature of 15000 K, and a wall temperature of 13000 K. Compute the electron density.*

Exercise 2 *Write a function which computes the electron heat conductivity as a function of electron density, heavy particle density, and electron density. Compute the electron heat conductivity at each point, and write it to log. Check with the assistents that your value of the heat conductivity is correct.*

[1] In flowing plasmas, the momentum balance may also be quite important. A lot of expertise in solving the momentum balance has been acquired by fluid mechanics groups. Another potentially important process we ignore is radiation transport

Exercise 3 *Now, we will add the electron temperature equation. This is an equation like 5 in week 1. This equation needs three things: A linear source term* `sp` *(0.0 in this case), the transport coefficient* `gamma`*, computed in Exercise 2, and a constant source term* `sc`*. The latter depends on three things:*

- *Heat supplied to the plasma. This has been derived in an earlier exercise.*
- *Heat which is lost due to ionizations. This is basically the amount of particles created with ionization, which you computed in an earlier exercise, multiplied with the ionization energy.*
- *Heat which is gained due to recombination. This is the recombination rate, which you computed in in an earlier exercise, multiplied by the ionization energy.*

These source terms should be defined in the same loop where you define the source terms for the particle balance. You should then add a line to solve the electron temperature equation.

This is in principle enough to solve the system, although you should not forget to use the greater of the residues for determining when to terminate the loop. You should also write the electron temperature to log.

To ensure convergence, you may need to use underrelaxation. If this converges, congratulations! This is already a quite reasonable model of the cascaded arc. This is probably the most complex exercise, so feel free to ask for assistance.

1.4 Adding the heavy particle temperature

For many plasmas, the interaction between electron temperature and electron density is of paramount importance for a correct description. For some higher-density plasmas[2], the heavy particle temperature can get quite high, and a substantial amount of heat can be transported through gradients in the heavy particle temperature. Our system is no exception. We will add an equation for the heavy particle temperature in much the same way we added an equation for the electron temperature.

Exercise 4 *Add a function which computes the heavy particle heat conductivity as a function of the heavy particle density. You can use the same function you used week 1.*

Exercise 5 *Add a function which computes the electron-heavy particle energy transfer. You should linearize the function by splitting it in two parts: One that does not depend on T_h and one that depends on T_e. Your function should then compute $\nu_{ea}\frac{3m_e}{m_a}$. This should be subtracted from* `sp`*. By multiplying this with T_h, and adding it to* `sc`*, you take into account the constant part. Add this function to the code. Add an* `sc` *and and* `sp` *to the electron temperature source terms.*

[2] When a plasma physicist talks about high-density, he or she usually means high-electron density, and that usually means more than $1{\cdot}10^{21}$ m^{-3}.

Exercise 6 *Add a computation for the heavy particle temperature. In Exercises 4 and 5, you have made functions for the description of the coefficients of this calculations. You can use those. Don't forget to adjust the residue for the residue of the heavy particle calculation, and don't forget to use proper linearization for the electron-heavy energy transfer, analogous to Exercise 5Write the heavy particle temperature to screen. You will probably need to adjust the underrelaxation factors. You will probably notice that it is rather low. Try to think of a reason why (Hint: It's on the formula page).*

2 Coulomb-dominated plasmas

The results of Exercise 6 point to a very common mistake in modeling: to use invalid assumptions. The plasma is in a Coulomb-dominated regime. This means, that the long-range Coulomb interactions between electrons and ions are more important than the short-range electron-neutral interaction, even though there are more neutrals than ions. This is amplified by the fact that argon has a dip in the electron-neutral cross sections for electron energies of around 1 eV. This is due to quantummechanical effects. All noble gases exhibit this so-called Ramsauer minimum.

Having identified the problem, we can now try to solve it. There is an expression for the ion-electron collision frequency, given by:

$$\nu_{ei} = \frac{3.6 \cdot 10^{-6} n_e^2 \ln \Lambda}{(T_e)^{1.5}} \tag{1}$$

Here, ν_{ei} is the frequency for the collisions between all electrons and all ions, and $\ln \Lambda$ is the so-called Coulomb logarithm, which is about 10. The heat transfer between electrons and ions S_{ei} can now be approximated by:

$$S_{ei} = \nu_{ei} \frac{m_e}{m_h} k_b (T_e - T_h) \tag{2}$$

with m_h the heavy particle mass and m_e the electron mass.

Exercise 7 *Add the electron-ion heat transfer to the computation. The simplest way to do this is by making use of the similarities between the expression for the electron-neutral and the electron-ion transfer: both depend on $(T_e - T_h)$. It will be difficult to obtain convergence: You'll need to underrelax all three phi-variables. Make plots of the electron density, electron temperature, neutral density and heavy particle temperature.*

Exercise 8 *If you have time left, you can try to change some parameters, such as particle density, wall temperature, size, power, etc. and see what the results are.*