

1 Variables

Variable	Description	Value	Unit
π	Ratio of circle circumference to diameter	3.1416	–
k_B	Boltzmann's Constant	$1.38066 \cdot 10^{-23}$	JK ⁻¹
m_e	Electron mass	$9.11 \cdot 10^{-31}$	kg
h	Planck's constant	$6.62618 \cdot 10^{-34}$	J s
k_{ion}	Ionization rate		m ³ s ⁻¹
E_{ion}	Ionization energy		J
T_e	Electron temperature		K
T_h	Heavy particle temperature		K
m_h	Particle mass		kg
σ_{aa}	Heavy-heavy collision cross section		m ²
σ_{ia}	Ion-heavy collision cross section		m ²
λ_h	Heavy particle heat conductivity		JK ⁻¹ m ⁻¹ s ⁻¹
q	Exponent in Arrhenius		–
k_{rate}	Arrhenius reaction rate coefficient		m ³ s ⁻¹
k_{rec}	Recombination rate		m ³ s ⁻¹
k_{heat}	Heat transfer		J m ⁻³ s ⁻¹
G	Ratio of degeneracy of ion and ground state		–
n_a	Heavy particle density		m ⁻³
n_e	Electron density		m ⁻³
τ_{ia}	Ion-atom collision time		s
ν_{ea}	Volumetric electron-atom collision frequency		m ³ s ⁻¹
D_i	Ion diffusion coefficient		m ² s ⁻¹
D_{amb}	Ambipolar diffusion coefficient		m ² s ⁻¹
c_p	Isobaric heat capacity per cubic meter		J K ⁻¹ m ⁻³
c_v	Isochoric heat capacity per cubic meter		J K ⁻¹ m ⁻³

2 Equations

2.1 Low to moderate ionization (<1%)

Heat conductivity (Heavy particles)

$$\sqrt{\frac{8k_B T_h}{\pi m_h}} \frac{\sqrt{2}}{\sigma_{aa}} k_B \quad (1)$$

Ionization rate (Arrhenius, single-step; Mind the normalization of T_e in T_e^q)

$$k_{ion} = k_{rate} T_e^q \exp\left(\frac{-E_{ion}}{k_B T_e}\right) \quad (2)$$

Recombination rate

$$k_{rec} = \left(\frac{h}{\sqrt{2\pi m_e k_B T_e}}\right)^3 \frac{k_{rate}}{2G} T_e^q \quad (3)$$

Atom collision time for one ion

$$\tau_{ia} = \sqrt{\frac{\pi m_a}{8k_B T_h}} \frac{1}{\sigma_{ai} n_a} \quad (4)$$

Diffusion coefficient

$$D_i = \frac{k_B T_h}{M} \tau_{ia} \quad (5)$$

Ambipolar diffusion

$$D_{amb} = D_i \left(1 + \frac{T_e}{T_h}\right) \quad (6)$$

Collision frequency

$$\nu_{ea} = n_e n_a \sigma_{ea} \sqrt{\frac{8k_B T_e}{\pi m_e}} \quad (7)$$

Electron-heavy particle heat transfer

$$k_{heat} = \nu_{ea} \frac{3m_e}{m_a} k_B (T_e - T_h) \quad (8)$$

Isoobaric heat capacity of a single type of particle n :

$$c_p = \frac{5}{2} k_B n \quad (9)$$

Isochoric heat capacity of a single type of particle n :

$$c_v = \frac{3}{2} k_B n \quad (10)$$

warning:

The equations in this document are only approximately valid, and only for a limited parameter range, while containing undocumented assumptions. If you are looking for formulas for scientific use, it is suggested you look elsewhere.