Development of a consistent kinetic plasma model for the ionization layer in HID lamps using Hermite polynomials/Burnett functions

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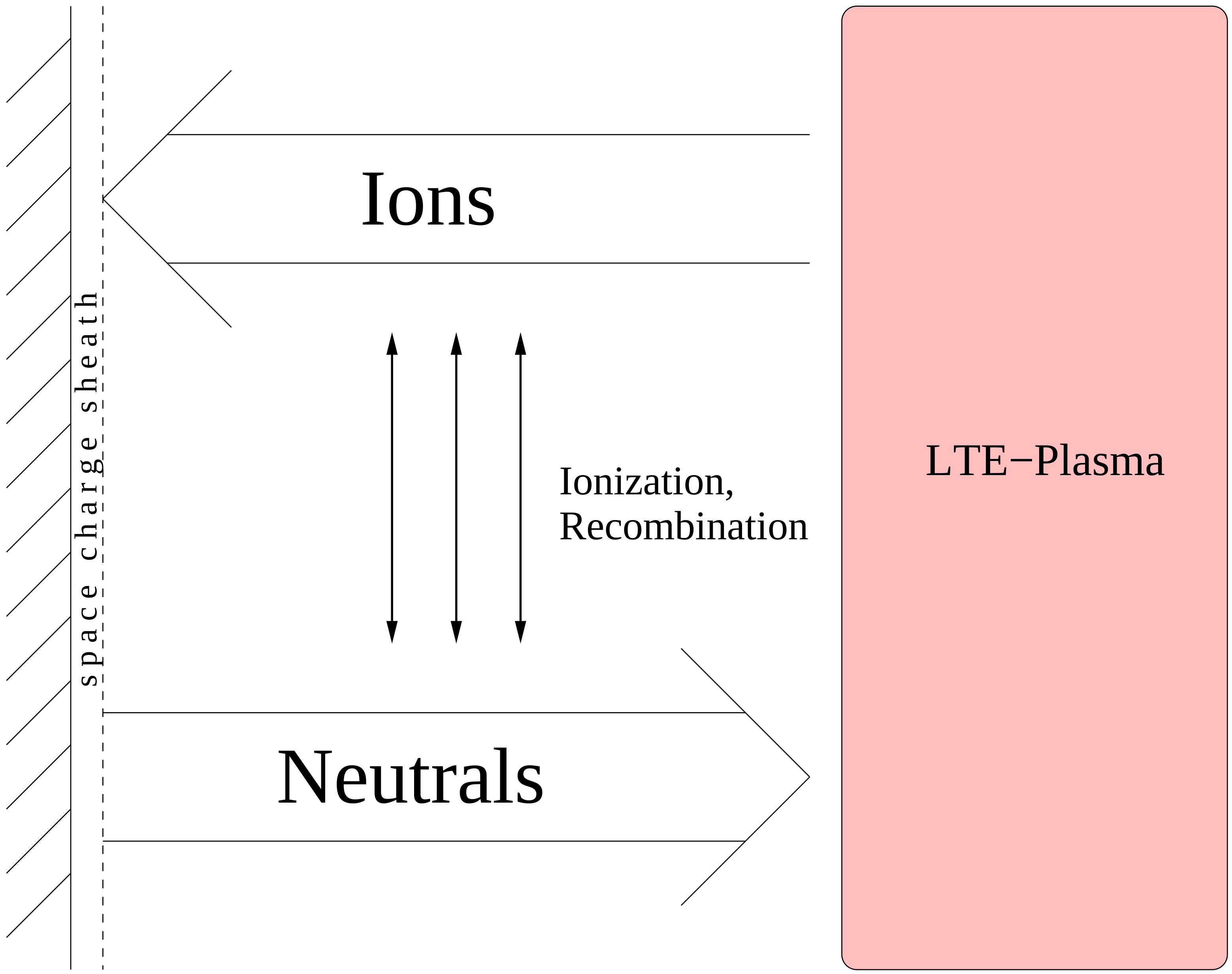
Introduction

- Experimental investigations of sheaths in HID lamps are complicated
- However, their properties are important
- Alternative: Modeling and simulation
- Majority of models is based on fluid dynamics: problematic
- This shows the need for a consistent kinetic model

Schematics



Ions
space charge sheath
Ionization,
Recombination
LTE−Plasma
Neutrals

Equations

- Focus on the heavy particle distribution functions $f_i,\ f_n$
- Boltzmann's equation:

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f + \underline{F}/m \cdot \nabla_v f =< f >_c$$

- Stationary model: $\frac{\partial f}{\partial t} = 0$
- One-dimensional in space: $\underline{r} \rightarrow z$
- Cylindric symmetry in velocity space: $\underline{v} \rightarrow \underline{v}_{\|} + \underline{v}_{\perp}$
- No magnetic field: $\underline{F} = e\underline{E}$
- Self-consistent electric field:

$$\underline{E} = -\frac{T_e}{e n_i(z)} \frac{\partial n_i(z)}{\partial z} \underline{e}_z$$

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Ion collision term $< f_i >_c$

Inelastic collisions:

- Ionization: $+\nu_i f_a$
- Recombination: $-\nu_r f_n$
- Charge exchange: $+\nu_{cx}(f_n - f_i n_{np}/n_{ip})$

Elastic collisions:

- Boltzmann collision term:

$$+ \int_{-\underline{\tilde{v}}} \int_{-\underline{\tilde{v}}'} \int_{-\underline{v}'} W_{ii} \left(f_i(\underline{\tilde{v}}') f_i(\underline{v}') - f_i(\underline{\tilde{v}}) f_i(\underline{v}) \right) \mathrm{d}\underline{v}' \, \mathrm{d}\underline{\tilde{v}}' \, \mathrm{d}\underline{\tilde{v}}$$

$$+ \int_{-\underline{\tilde{v}}} \int_{-\underline{\tilde{v}}'} \int_{-\underline{v}'} W_{in} \left(f_n(\underline{\tilde{v}}') f_i(\underline{v}') - f_n(\underline{\tilde{v}}) f_i(\underline{v}) \right) \mathrm{d}\underline{v}' \, \mathrm{d}\underline{\tilde{v}}' \, \mathrm{d}\underline{\tilde{v}}$$

Neutral collision term $< f_n >_c$

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Inelastic collisions:

- Ionization: $-\nu_i f_n$
- Recombination: $+\nu_r f_n$
- Charge exchange: $-\nu_{cx}(f_n n_{ip}/n_{np} - f_i)$

Elastic collisions:

- Boltzmann collision term:

$$+\int_{-\underline{\tilde{v}}}\int_{-\underline{\tilde{v}}'}\int_{-\underline{v}'} W_n \left(f_n(\underline{\tilde{v}}')\, f_n(\underline{v}') - f_n(\underline{\tilde{v}})\, f_n(\underline{v}) \right) \mathrm{d}\underline{v}'\, \mathrm{d}\underline{\tilde{v}}'\, \mathrm{d}\underline{\tilde{v}}$$

$$+\int_{-\underline{\tilde{v}}}\int_{-\underline{\tilde{v}}'}\int_{-\underline{v}'} W_{ni} \left(f_i(\underline{\tilde{v}}')\, f_n(\underline{v}') - f_i(\underline{\tilde{v}})\, f_n(\underline{v}) \right) \mathrm{d}\underline{v}'\, \mathrm{d}\underline{\tilde{v}}'\, \mathrm{d}\underline{\tilde{v}}$$

Ansatz

- Basic idea: Maxwell distribution + Deviation (deviation not necessarily small!)

$$f(z,\underline{v}) = n\left(\frac{m}{2\pi T_h}\right)^{3/2} e^{-m\underline{v}^2/(2T_h)}\left(1 + X(z,\underline{v})\right)$$

- At the LTE-Plasma, the deviation vanishes:

$$\lim_{z\to\infty}(X) = 0$$

- The deviation is written in terms of orthogonal functions:

$$X(z,\underline{v}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} a_{ijk}(z)\mathrm{He}_i(v_x)\mathrm{He}_j(v_y)\mathrm{He}_k(v_z)$$

$$= \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} a_{ijk}(z)\mathrm{He}_i(v_x)\mathrm{He}_j(v_y)\mathrm{He}_k(v_z)$$

Pros and Cons

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- Expansion around zero
- Advantage:
  simpler equations, since calculation of drift velocity is not necessary
- Disadvantage:
  converges slower, more coefficients are needed

- How important is this disadvantage?

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Reminder: Hermite polynomials

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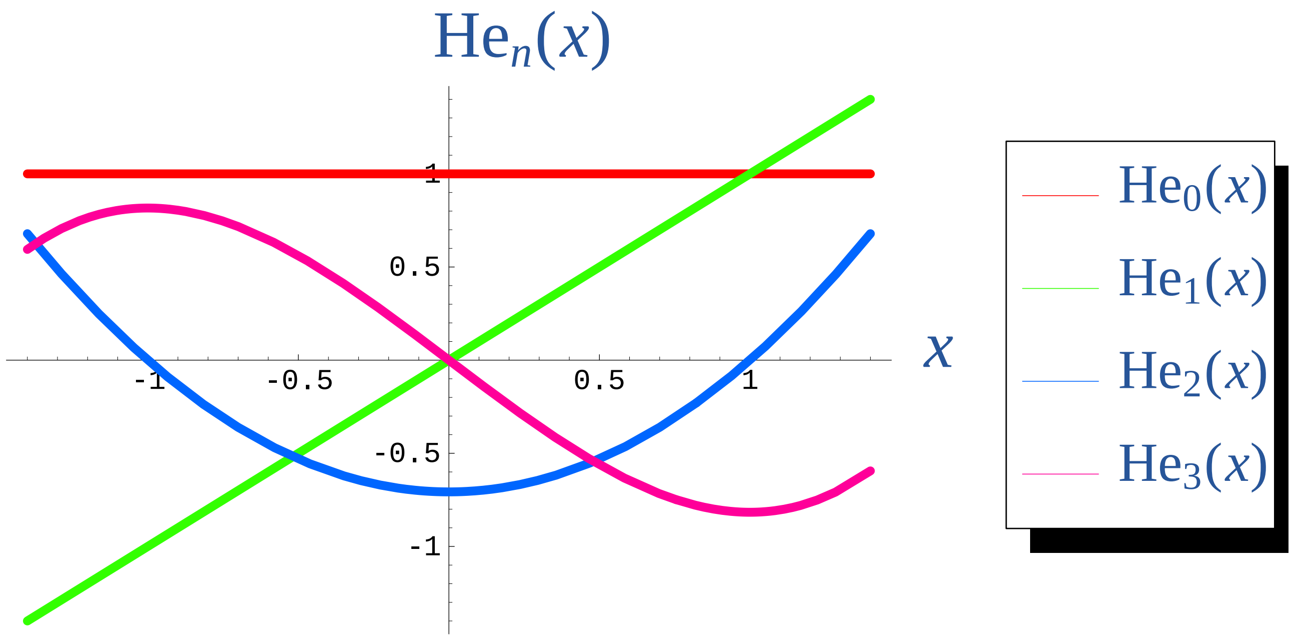

$$\mathrm{He}_n(x) = \frac{2^{-n/2}}{\sqrt{n!}} e^{x^2/2} \left[\left(\frac{\partial}{\partial t} \right)^n e^{-(x/\sqrt{2}-t)^2} \right]_{t=0}$$

- Satisfy the differential equation $y'' - xy' + ny = 0$
- Orthogonal polynomials with weight function $(2\pi)^{-1/2} e^{-x^2/2}$ in the interval $(-\infty, \infty)$

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Hermite Polynomials orthogonal series

- Orthogonal polynomials → orthogonal series possible

$$f(v) = \sum_{n=0}^{\infty} a_n \mathrm{He}_n(v) \frac{e^{-v^2/2}}{\sqrt{2\pi}} \approx \sum_{n=0}^{N} a_n \mathrm{He}_n(v) \frac{e^{-v^2/2}}{\sqrt{2\pi}}$$

$$\text{with} \quad a_n = \int_{-\infty}^{\infty} f(v) \mathrm{He}_n(v) dv$$

- Converges relatively fast for Gaussian-like functions
- Converges (although more slowly) for small shifts of such functions

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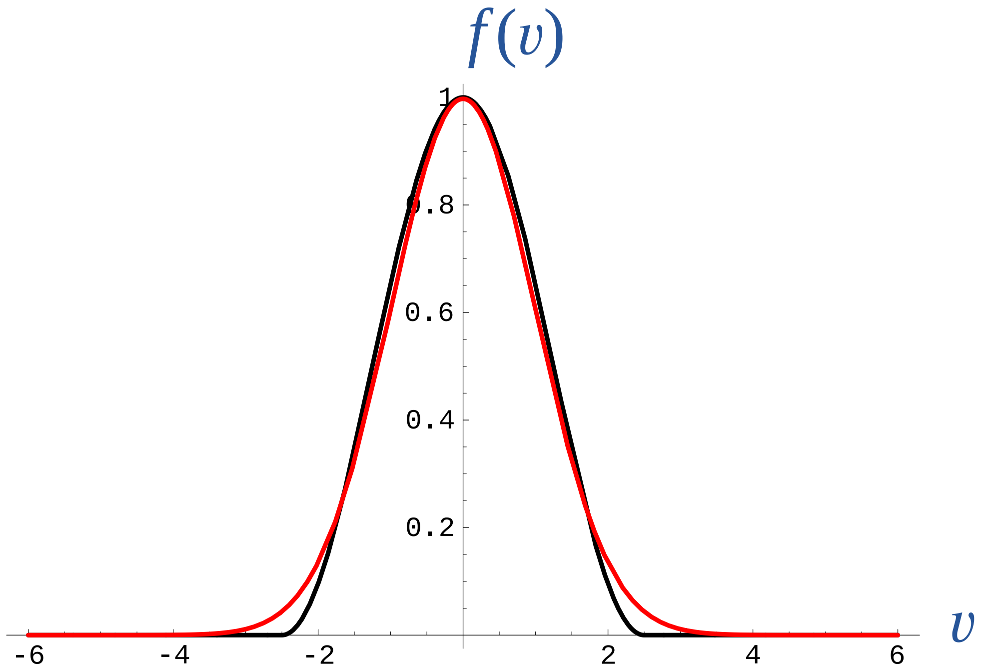
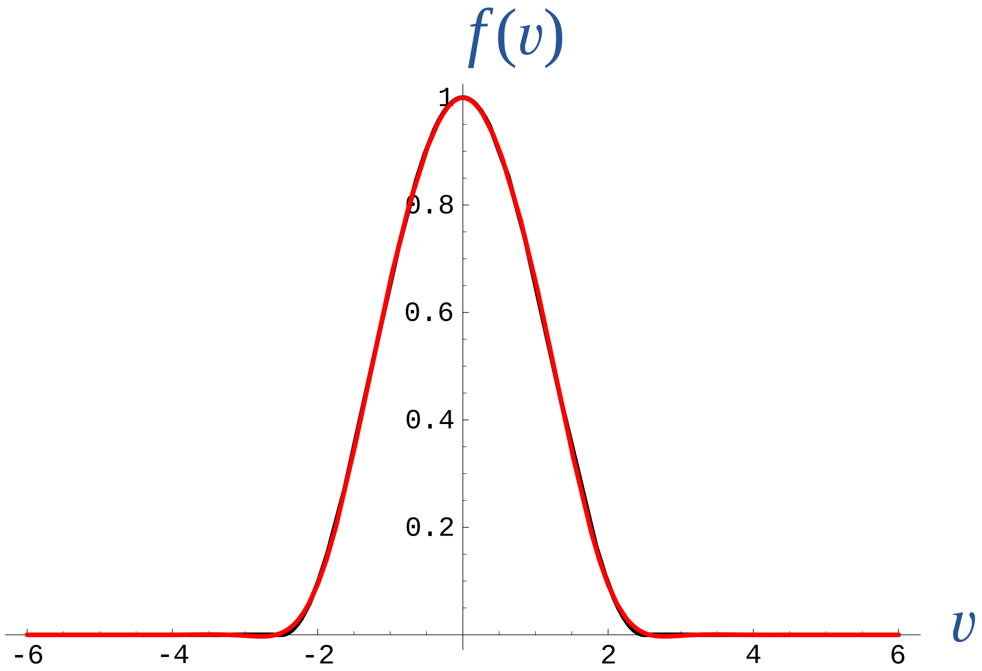
Convergence: 1D, good

- $N = 1:$

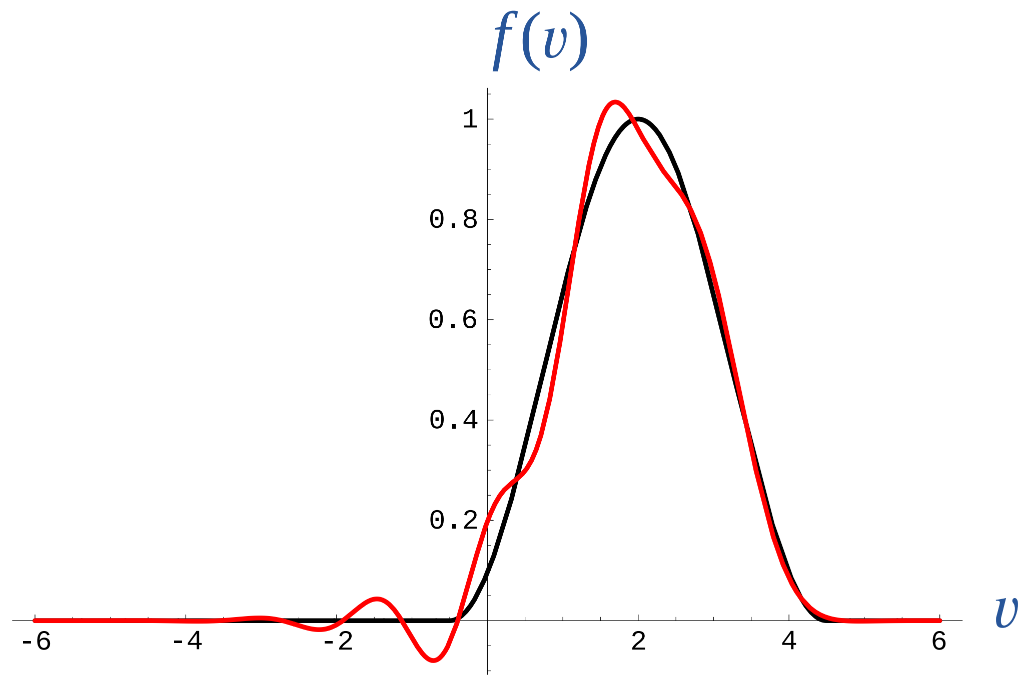


- $N = 17:$

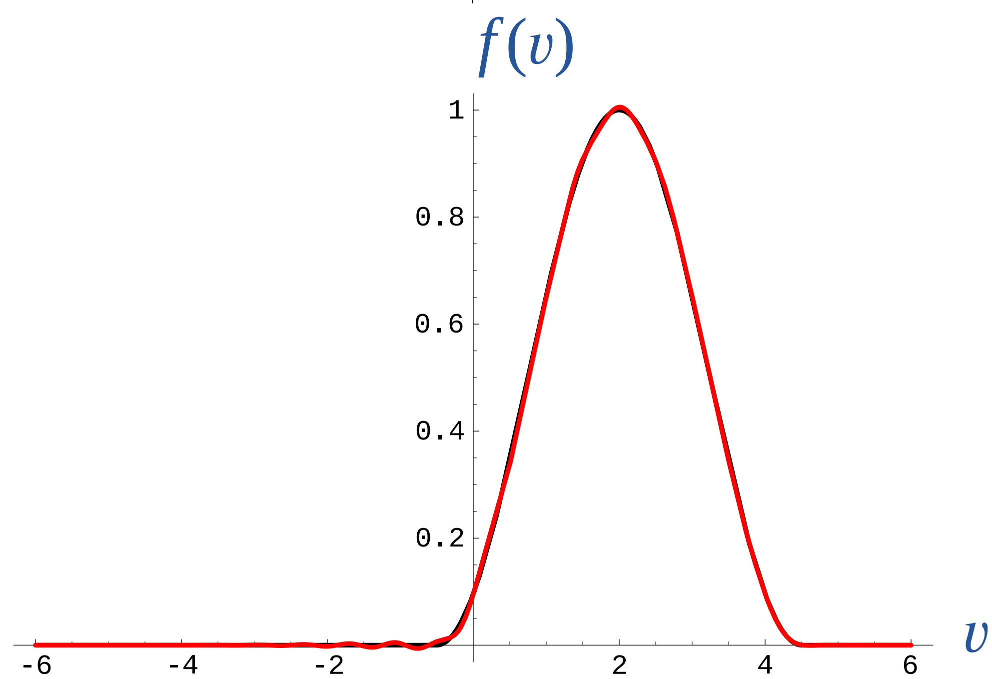


Convergence: 1D, still acceptable

- $N = 17:$



- $N = 100:$



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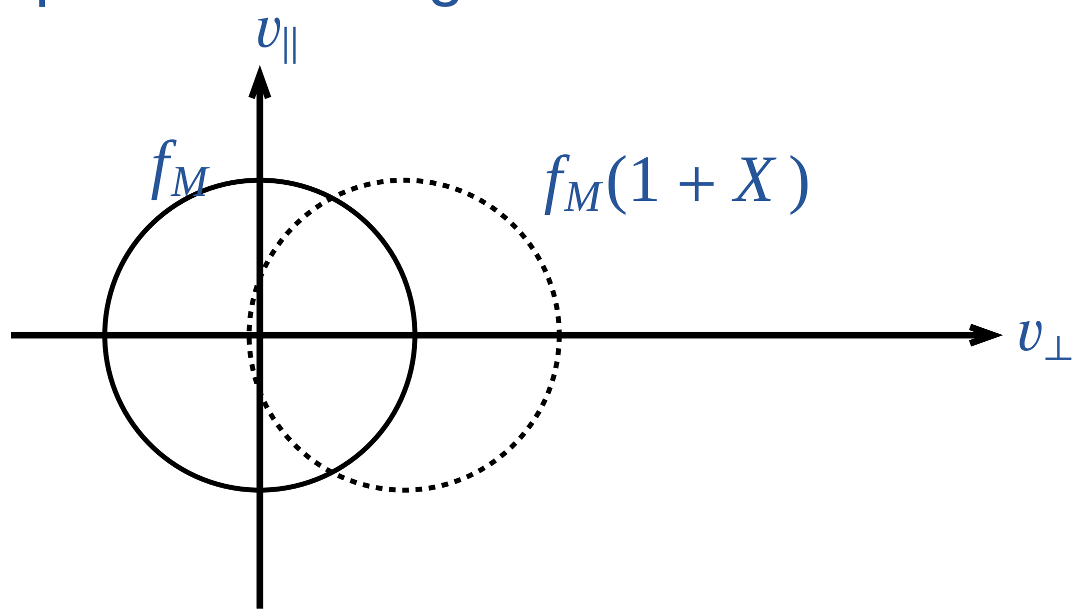
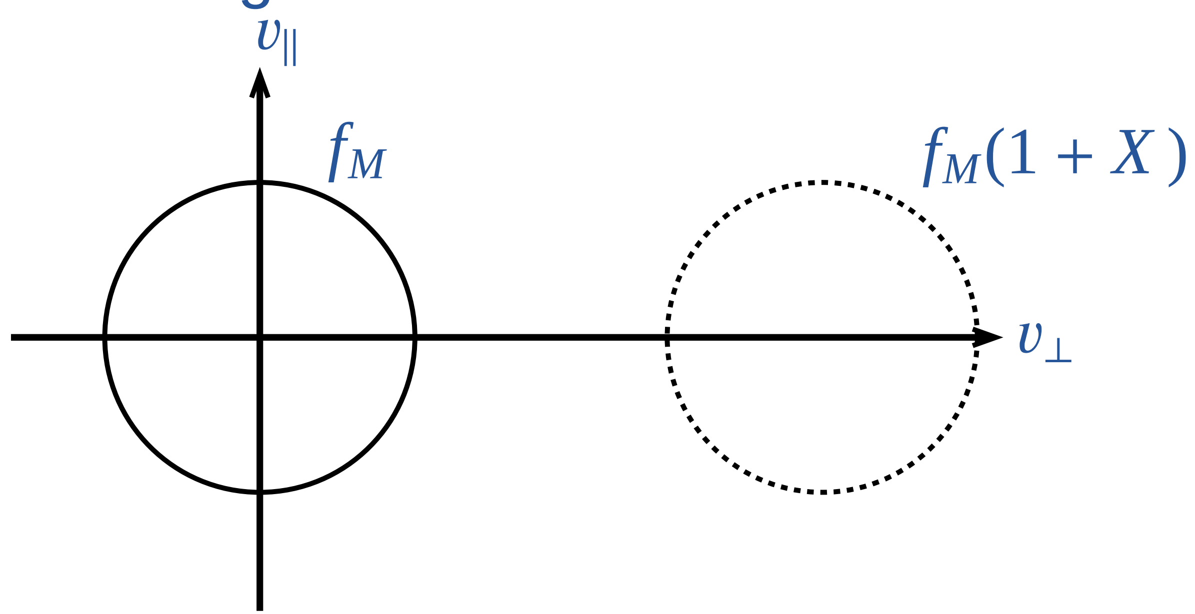
Convergence: 2D

Case I: Acceptable convergence



Case II: Bad convergence



Convergence: summary

- At the point of largest deviation X: Bohm velocity
- Energy is about equal to the thermal energy of the electrons
- Drift velocity is of the same order as v_{th}
- Distribution function shift is small

→ We expect acceptable convergence!

- Agrees with values calculated from fluid models

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Further Simplifications

- Further simplifications of the orthogonal series:

$$X(z,\underline{v}) = \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} a_{ijk}(z)\mathrm{He}_i(v_x)\mathrm{He}_j(v_y)\mathrm{He}_k(v_z)$$

$$= \sum_{n=0}^{N}\sum_{l=0}^{L}\sum_{m=-l}^{l} a_{nlm}(z)\mathrm{B}_{nlm}(\varepsilon,\vartheta,\varphi)$$

$$= \sum_{n=0}^{N}\sum_{l=0}^{L}\sum_{m=-l}^{l} a_{nlm}(z)\varepsilon^{l/2}\mathrm{L}_n^{l+1/2}(\varepsilon)\mathrm{Y}_{lm}(\vartheta,\varphi)$$

$$= \sum_{n=0}^{N}\sum_{l=0}^{L} a_{nl0}(z)\varepsilon^{l/2}\mathrm{L}_n^{l+1/2}(\varepsilon)\mathrm{Y}_{l0}(\vartheta)$$

- Calculation of coefficients by applying orthogonality criterion (lots of lots of differential equations....)

Summary and Outlook

In this talk:

- Introduction to the model
- Hermite Polynomials
- Convergence considerations

In the future:

- Compose a detailed model
- Determine the coefficients
- Compare the model results with experiments

Thank you!

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